MATH 406 – HOMEWORK 4

(due Monday 12 March 2012)

1. Find all solutions to $15x \equiv 9 \pmod{24}$.

2. Determine all integers (between 1 and 20) which are their own inverses modulo 20. Explain.

3. Let p be an odd prime. Let a be a positive integer not divisible by p. Assume that $x^2 \equiv a \pmod{\text{p}}$ has a solution. Show that it has exactly two incongruent solutions.

4. Find all solutions to the following system of congruences.

$x \equiv 1 \pmod{2}$

$x \equiv 2 \pmod{3}$

$x \equiv 3 \pmod{5}$

5. Find the smallest positive integer which is divisible by 13 but leaves a remainder of 9 when divided by either 10 or 11.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.