MATH 406 – HOMEWORK 5

(due Monday 26 March 2012)

1. Determine the remainder when $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12 \cdot 14 \cdot 16 \cdot 18 \cdot 20 \cdot 22 \cdot 24$ is divided by 13. Explain.

2. Determine the remainder when 6^{2009} is divided by 11. Explain.

3. Find the last digit of the decimal expansion of $7^{1,000,001}$.

4. Use Euler's Theorem to solve $5x \equiv 3 \pmod{16}$.

5. Let a, b be positive integers with $(a, b) = 1$. Show that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.