MATH 406 – HOMEWORK 6

(due Monday 2 April 2012)

1. Determine all positive integers n with $\phi(n) = 14$. Explain.

2. Show that $\phi(m^k) = m^{k-1}\phi(m)$ for all positive integers m and k.

3. Assume that $n \in \mathbb{Z}^+$ has k distinct odd prime divisors. Prove that $2^k | \phi(n)$.

4. Determine all positive integers n for which $\sigma(n)$ is odd. Explain.

5. Let $m \geq 2$, let $n = 2^{m-1}(2^m - 1)$, and assume that $2^m - 1$ is not prime. Show that $\sigma(n) > 2n$.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.