MATH 406 – HOMEWORK 8

(due Monday 30 April 2012)

1. [6 pts] How many incongruent roots does x^3+12 have modulo 13? Explain carefully.

2. [6 pts] Prove Euler's Criterion (Theorem 11.3) using indices and Theorem 11.2.

3. [6 pts] Use Gauss' Lemma to evaluate $\left(\frac{5}{17}\right)$.

4. [7 pts] Let p be a prime ≥ 7.

(a) Prove that at least one of 2, 5, and 10 is a quadratic residue of p.

(b) Conclude that there is some $a \in \mathbb{Z}$ such that both a and $a+1$ are quadratic residues of p.

5. [5 pts] Use the arithmetic properties of the Legendre symbol to evaluate $\left(\frac{-8}{17}\right)$.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.