MATH 406 – HOMEWORK 9

(due Monday 7 May 2012)

1. Use the arithmetic properties of the Legendre symbol to evaluate $\left(\frac{77}{103}\right)$.

2. Let p and q be odd primes and assume that $p \equiv q \pmod{20}$. Prove that $\left(\frac{5}{p}\right) = \left(\frac{5}{q}\right)$.

3. Show that if x, y, z is a primitive Pythagorean triple then either x or y is divisible by 3.

4. Find formulas for the integers of all (primitive) Pythagorean triples x, y, z with $z = y + 1$.

5. Show that the Diophantine equation $x^4 + 3y^4 = z^2$ has infinitely many solutions.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.