MATH 445 – Exam I – 1 October 2010

Work each numbered problem on a separate answer sheet. Show all your work for each problem clearly on the answer sheet for that problem. Write your name and the problem number on each answer sheet. Good luck.

1. [15 pts] Use the equivalences in the notes (*not* truth tables) to find both the CNF and the DNF of the following sentence:

$$\big(A \to (B \to C)\big) \vee \neg(B \to D)$$

Simplify your answers if possible.

2. Determine which of the following are valid and which are satisfiable but not valid. Explain – in particular for those which are satisfiable but not valid give both an assignment satisfying the sentence and an assignment falsifying it.

[11 pts] (a) $\big((C \to A) \to B\big) \to (C \to B)$

[11 pts] (b) $\big((A \to C) \to B\big) \to \big((A \to B) \to B\big)$

[11 pts] (c) $\big(A \to \neg(B \to C)\big) \vee \big(B \to \neg(A \to C)\big)$

3. [16 pts] Show the following. In addition to the axioms and Modus Ponens, you may use the Deduction Theorem and the facts that $\vdash (\varphi \to \varphi)$, $\vdash (\neg\varphi \to \varphi) \to \varphi$, and $\vdash \varphi \to (\neg\varphi \to \psi)$, but no other derived facts.

$$\vdash (\neg A \to \neg B) \to (B \to A)$$

4. Either prove or disprove (with a counterexample) each of the following:

[7 pts] (a) $\neg\varphi$ is satisfiable iff φ is not valid.

[7 pts] (b) If $(\varphi \to \psi)$ is valid then ψ is satisfiable.

[7 pts] (c) If the set Σ is satisfiable and $\Sigma \cup \{\varphi\}$ is not satisfiable then $\Sigma \cup \{\neg\varphi\}$ is satisfiable.

5. [5 pts] (a) Define what it means for a set Σ to be *consistent*.

[5 pts] (b) Define what it means for Σ to be *maximal consistent*.

[5 pts] (c) Assume that Σ is maximal consistent. Prove, directly from the definitions, that if $\Sigma \vdash \varphi$ then $\varphi \in \Sigma$.

NOTE: Your solutions must include enough detail to justify your conclusions.