MATH 445 — Exam 1 Review

Topics

- sentences, truth assignments, and satisfaction
- validity, satisfiability, and the relation between them
- truth tables and other ways of determining validity and satisfiability
- logical consequence and its relation to satisfiability
- ways of determining logical consequence
- equivalence, rewriting, DNF, CNF
- logical axioms, MP, deductions, and Soundness
- deductions from hypotheses, the Deduction Theorem, Soundness
- consistent sets of sentences: definition and characterization
- the relation between deducibility and consistency
- extending consistent sets, maximal consistent sets
- consistent sets are satisfiable, Completeness Theorem
- Compactness Theorem

Sample Problems

1. Assume that $h \not\models (A \vee B) \to (A \wedge B)$. What can you conclude about $h(A)$ and $h(B)$? Explain carefully.

2. Write down the complete truth tables for each of the following:

(a) $\big((A \to B) \to B\big) \to B$

(b) $\big((A \to B) \vee (B \to C)\big) \to (B \vee C)$

3. Use the equivalences given in the notes (not truthtables) to find the DNF and CNF of $(A \to \neg C) \to (B \to \neg C)$.

4. Determine which of the following are valid and which are satisfiable but not valid. Explain – in particular for those which are satisfiable but not valid you must give both an assignment satisfying the sentence and an assignment falsifying it.

(a) $\big((A \to B) \to C\big) \to (A \to C)$

(b) $\big((A \to B) \to C\big) \to (\neg A \to C)$

(c) $(A \to C) \to \big((A \to B) \to C\big)$

(d) $(\neg A \to C) \to \big(A \to B) \to C\big)$

5. Prove or disprove (with a counterexample) each of the following:

(a) If both φ and ψ are satisfiable then so is $\varphi \wedge \psi$.

(b) If $\varphi \models \psi$ then $\models (\varphi \to \psi)$.

(c) If $\models (\varphi \to \psi)$ then ψ is satisfiable.

(d) If $\varphi \models \theta$ and $\psi \models \theta$ then $(\varphi \vee \psi) \models \theta$.

6. Establish the following. You may use the Deduction Theorem and the facts that $\vdash \varphi \to \varphi$, $\vdash (\neg\varphi \to \varphi) \to \varphi$, and $\vdash \varphi \to (\neg\varphi \to \psi)$ but no other derived facts.

(a) $\vdash \big((\varphi \to \psi) \to (\psi \to \theta)\big) \to (\psi \to \theta)$

(b) $\vdash (\neg\varphi \to \psi) \to (\neg\psi \to \varphi)$

7. (a) Define what it means for a set Σ to be *consistent*.

(b) Prove that if Σ is inconsistent then $\Sigma \vdash \psi$ for every sentence ψ.

(c) Define what it means for Σ to be *maximal consistent*.

8. Prove or disprove (with a counterexample) each of the following:

(a) If $\Sigma \cup \{\varphi\}$ is inconsistent then $\Sigma \cup \{\neg\varphi\}$ is consistent.

(b) If $\Sigma \cup \{\varphi\}$ is consistent then $\Sigma \cup \{\neg\varphi\}$ is inconsistent.

9. (a) State the Completeness Theorem.

(b) State the Compactness Theorem.

(c) Prove the Compactness Theorem, using the Completeness Theorem.