MATH 445 – Homework 1

(due Friday 11 September 2009)

1. Let α be the following expression.

$$((A \wedge \neg B \vee (C \wedge D) \vee E)$$

[1 pt] (a) Show that α is not a sentence.

[3 pts] (b) Add parentheses to α in three different ways so that the result is a sentence.

2. Assume that $h \models (A \to B) \to C$. Determine which of the following h *must* satisfy — explain carefully.

[2 pts] (a) $A \to C$

[2 pts] (b) $\neg A \to C$

3. [4 pts] Write down the complete truth table for the following sentence.

$$(A \to (B \vee C)) \to (C \to \neg\,(B \to A))$$

4. [4 pts] Determine some truth assignment satisfying

$$\{(C \to B) \to (A \to \neg D), (B \to C) \to (D \to A), \neg(B \to \neg D)\}$$

without using truth tables. You must explain both *how* you obtained the assignment and *why* it works.

5. [4 pts] We define a sequence of sentences as follows:
$\varphi_1 = S_1, \varphi_2 = S_1 \to S_2, \varphi_3 = (S_1 \to S_2) \to S_3$, in general $\varphi_{n+1} = \varphi_n \to S_{n+1}$.
Define $h(S_k) = F$ for all k. Prove that $h \models \varphi_n$ iff n is even.

NOTE: Your solutions must include enough detail to justify your conclusions.