MATH 445 – Homework 1

(*due Friday 10 September 2010*)

1. [5 pts] Let α be the following expression.

$$((A \to B \to \neg C \to D))$$

α is not a sentence, but it can be made into a sentence by adding pairs of parentheses.

(a) How many pairs of parentheses must be added to make α into a sentence? Explain.

(b) Write down all of the sentences which can be obtained from α in this way.

2. [5 pts] Assume that $h \models (A \to B) \to \neg(B \to C)$. Determine which of the following h *must* satisfy — explain carefully.

(a) $B \to \neg C$

(b) $A \to \neg C$

3. [5 pts] Write down the complete truth table for the following sentence.

$$\big((A \to B) \vee (B \to C)\big) \to (A \to C)$$

4. [5 pts] We define a sequence φ_n of sentences as follows:

$$\varphi_n = (S_n \to S_{n+1}) \text{ for all } n \in \mathbb{N}.$$

Prove that for every $n \in \mathbb{N}$ there is an interpretation h_n such that

$$h_n \models \varphi_k \text{ for all } k \neq n \text{ but } h_n \not\models \varphi_n.$$

NOTE: Your solutions must include enough detail to justify your conclusions.