MATH 445 – Homework 1

(*due Monday 12 September 2011*)

1. [5 pts] Let α be the following expression.

$$\neg A \to B \to C$$

α is not a sentence, but it can be made into a sentence by adding parentheses. Write down all of the sentences which can be obtained from α by adding (only) parentheses. Explain briefly.

2. [5 pts] Assume that $h \models \big((A \to B) \to C\big)$. Determine which of the following h *must* satisfy — explain carefully.

(a) $(A \to C)$

(b) $(\neg A \to C)$

3. [5 pts] Write down the complete truth table for the following sentence.

$$\big((A \to (B \vee C)\big) \to \big(C \to \neg(A \to B)\big)$$

4. [5 pts] We define a sequence φ_n, for $n \in \mathbb{N}$, of sentences as follows:

$$\varphi_n = (S_{n+1} \to \neg S_n) \text{ for all } n \in \mathbb{N}.$$

Prove that for every $n \in \mathbb{N}$ there is an interpretation h_n such that

$$h_n \models \varphi_k \text{ for all } k \neq n \text{ but } h_n \not\models \varphi_n.$$

NOTE: Your solutions must include enough detail to justify your conclusions.