MATH 445 – Homework 2

(*due Friday 18 September 2009*)

1. [5 pts] Determine which of the following are valid, which are satisfiable but not valid, and which are contradictions. Justify your answers – in particular if you claim a sentence is satisfiable but not valid you must give both an assignment satisfying it and an assignment falsifying it.

(a) $\big((A \to B) \to (B \to C)\big) \to (A \to C)$

(b) $\big(B \to (C \to A)\big) \vee \big(C \to (A \to B)\big)$

(c) $(\neg A \to B) \wedge \neg(C \to A) \wedge \neg(C \to \neg B)$

(d) $\neg(B \to A) \wedge (C \to A) \wedge (B \to C)$

2. Prove or disprove (with a counterexample) each of the following:

[2 pts] (a) If $\Sigma \models \varphi$ or $\Sigma \models \psi$ then $\Sigma \models (\varphi \vee \psi)$.

[2 pts] (b) If $\Sigma \models (\varphi \vee \psi)$ then either $\Sigma \models \varphi$ or $\Sigma \models \psi$.

3. [3 pts] Use the equivalences given in the notes (*not* truth tables) to find a sentence in CNF equivalent to the following sentence. Simplify if possible. Show all your work.

$$\big(D \to (A \wedge \neg C)\big) \vee (\neg A \to B)$$

4. Consider the following truth table.

A	B	C	θ
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

[2 pts] (a) Find a sentence in DNF with the above truth table. Justify your answer.

[2 pts] (b) Find a sentence in CNF with the above truth table. Justify your answer.

Definition. A set Σ is *independent* provided there is no $\varphi \in \Sigma$ such that $(\Sigma \setminus \{\varphi\}) \models \varphi$.

5. [4 pts] Let $\Sigma = \{(S_{i+1} \to S_i) : i \in \mathbb{N}\}$. Either prove that Σ is independent or prove that it is not independent.

NOTE: Your solutions must include enough detail to justify your conclusions.