MATH 445 – Homework 2

(*due Monday 20 September 2010*)

1. [6 pts] Determine which of the following are valid, which are satisfiable but not valid, and which are contradictions. Justify your answers – in particular if you claim a sentence is satisfiable but not valid you must give both an assignment satisfying it and an assignment falsifying it.

(a) $(A \to B) \to \big((C \to B) \to (C \to A)\big)$

(b) $\big((C \to B) \to (A \to C)\big) \to (A \to C)$

(c) $\big((A \to B) \to C\big) \wedge \big((B \to C) \to \neg A\big) \wedge \big((A \to B) \to \neg C\big)$

2. [4 pts] Prove or disprove (with a counterexample) each of the following:

[2 pts] (a) If $\Sigma \models \varphi$ or $\Sigma \models \psi$ then $\Sigma \models (\varphi \vee \psi)$.

[2 pts] (b) If $\Sigma \models (\varphi \vee \psi)$ then either $\Sigma \models \varphi$ or $\Sigma \models \psi$.

3. [5 pts] Use the equivalences given in the notes (*not* truth tables) to find both

(a) a sentence in DNF and

(b) a sentence in CNF

equivalent to the following sentence. Simplify if possible. Show all your work.

$$\big(B \to \neg(C \to A)\big) \wedge \neg(C \to D)$$

4. [5 pts] Consider the following truth table.

A	B	C	θ
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

(a) Find a sentence in DNF with the above truth table. Justify your answer.

(b) Find a sentence in CNF with the above truth table. Justify your answer.

NOTE: Your solutions must include enough detail to justify your conclusions.