MATH 445 – Homework 3

(*due Monday 27 September 2010*)

1. Prove or disprove (with a counterexample) each of the following:

[2 pts] (a) If $\varphi \vee \psi \models \theta$ then either $\varphi \models \theta$ or $\psi \models \theta$.

[2 pts] (b) If either $\varphi \models \theta$ or $\psi \models \theta$ then $\varphi \vee \psi \models \theta$.

2. [4 pts] Assume that $\Sigma \vdash \psi$ and $\Sigma \vdash (\neg\varphi \to \neg\psi)$. Show DIRECTLY (i.e., not using the Deduction Theorem or other results) that $\Sigma \vdash \varphi$.

3. Show all of the following. In addition to the logical axioms and Modus Ponens, you may use the Deduction Theorem and the known results $\vdash \varphi \to \varphi$ and $\vdash (\neg\varphi \to \varphi) \to \varphi$ but no other facts.

[4 pts] (a) $\vdash \big((\varphi \to \psi) \to (\psi \to \theta)\big) \to (\psi \to \theta)$

[4 pts] (b) $\vdash \neg\varphi \to (\varphi \to \psi)$

[4 pts] (c) $\vdash \neg\neg\varphi \to \varphi$

NOTE: Your solutions must include enough detail to justify your conclusions.