MATH 445 – Homework 5

(*due Monday 26 October 2009*)

1. [5 pts] Let $\mathcal{L}$ be the language whose non-logical symbols are a binary relation symbol R and two unary relation symbols E and P. Let $\mathcal{A}$ be the $\mathcal{L}$-structure with universe $\mathbb{N}$ which interprets R by $\leq$, interprets E by the set of even numbers, and interprets P by the set of prime numbers. Give sentences of $\mathcal{L}$ which 'naturally' express the following facts about $\mathcal{A}$.

(a) There is a smallest prime number.

(b) There is no largest prime number.

(c) There is exactly one even prime number.

2. [3 pts] Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary relation symbol R. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, <)$ and let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{Q}, <)$. Find a sentence σ of $\mathcal{L}$ such that $\mathcal{A} \models \sigma$ but $\mathcal{B} \models \neg\sigma$. Explain why your sentence works.

3. Working directly from the definitions, either prove or find a counterexample to each of the following:

[3 pts] (a) $\models (\exists x P(x) \to \exists x Q(x)) \to \exists x (P(x) \to Q(x))$

[3 pts] (b) $\models \exists x (P(x) \to Q(x)) \to (\exists x P(x) \to \exists x Q(x))$

[3 pts] (c) $\models \forall y \exists x R(x, y) \to \exists x \forall y R(x, y)$

[3 pts] (d) $\models \exists x \forall y R(x, y) \to \forall y \exists x R(x, y)$

NOTE: Your solutions must include enough detail to justify your conclusions.