MATH 445 – Homework 5

(*due Monday 25 October 2010*)

1. [5 pts] Let $\mathcal{L}$ be the language whose non-logical symbols are a binary relation symbol R and a constant c. Let $\mathcal{A}$ be the $\mathcal{L}$-structure with universe $\mathbb{Z}$ which interprets R by $\leq$ and interprets c by 0.

(a) Give an $\mathcal{L}$- formula $\varphi(x)$ which defines $\{-1\}$ in $\mathcal{A}$.

(b) Give an $\mathcal{L}$- formula $\psi(x)$ whhich defines $\{2\}$ in $\mathcal{A}$.

Explain why your formulas work.

2. [3 pts] Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary function symbol F. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{N}, +)$ and let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, +)$. Find a sentence σ of $\mathcal{L}$ such that $\mathcal{A} \models \sigma$ but $\mathcal{B} \models \neg\sigma$. Explain why your sentence works.

3. Working directly from the definitions, either prove or find a counterexample to each of the following:

[3 pts] (a) $\models (\forall x P(x) \to \forall x Q(x)) \to \exists x (P(x) \to Q(x))$

[3 pts] (b) $\models \exists x (P(x) \to Q(x)) \to (\forall x P(x) \to \forall x Q(x))$

[3 pts] (c) $\models \forall y \exists x R(x, y) \to \exists x \forall y R(x, y)$

[3 pts] (d) $\models \exists x \forall y R(x, y) \to \forall y \exists x R(x, y)$

NOTE: Your solutions must include enough detail to justify your conclusions.