MATH 445 – Homework 5

(*due Monday 24 October 2011*)

1. [5 pts] Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary relation symbol R. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{N}, \leq)$.

(a) Give an $\mathcal{L}$-formula $\varphi(x)$ which defines $\{0\}$ in $\mathcal{A}$.

(b) Give an $\mathcal{L}$-formula $\psi(x)$ which defines $\{2\}$ in $\mathcal{A}$.

Explain why your formulas work.

2. [3 pts] (a) Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary relation symbol R. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, <)$ and let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{Q}, <)$. Find a sentence σ of $\mathcal{L}$ such that $\mathcal{A} \models \sigma$ but $\mathcal{B} \models \neg\sigma$.

(b) [3 pts] Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary function symbol F. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{N}, +)$ and let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, +)$. Find a sentence σ of $\mathcal{L}$ such that $\mathcal{A} \models \sigma$ but $\mathcal{B} \models \neg\sigma$. Explain why your sentence works.

3. Working directly from the definitions, either prove or find a counterexample to each of the following:

[3 pts] (a) $\models (\exists x P(x) \to \exists x Q(x)) \to \exists x (P(x) \to Q(x))$

[3 pts] (b) $\models \exists x (P(x) \to Q(x)) \to (\exists x P(x) \to \exists x Q(x))$

[3 pts] (c) $\models \forall y (P(y) \to \exists x P(x))$

NOTE: Your solutions must include enough detail to justify your conclusions.