MATH 445 – Homework 6

(*due Monday 1 November 2010*)

1. Using the rules in the notes (but not, of course, the Completeness Theorem):

[4 pts] (a) show that $\vdash (\forall x P(x) \to \exists x Q(x)) \to \exists x(P(x) \to Q(x))$.

[4 pts] (b) assuming that $\Sigma \vdash (\varphi \to \psi)$ where Σ is a set of sentences, show that $\Sigma \vdash \forall x(\varphi \to \psi)$.

2. Let $\mathcal{L}$ be the language whose only non-logical symbols are a binary relation symbol R and a constant c. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, \leq, 0)$.

[3 pts] (a) Give a first-order sentence naturally expressing the following fact about $\mathcal{A}$: "The immediate predecessor of any positive integer is non-negative."

[3 pts] (b) Give a formula $\varphi(x, y)$ of $\mathcal{L}$ such that $\mathcal{A} \models \varphi(k, l)$ iff $l = k + 2$. Explain why your formula works.

3. Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary function symbol F. Let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{N}, +)$.

[3 pts] (a) Give a formula $\psi(x)$ of $\mathcal{L}$ which defines the set of numbers which are divisible by 3 but not by 2. Explain why your formula works.

[3 pts] (b) Give a formula $\chi(x, y)$ of $\mathcal{L}$ such that $\mathcal{B} \models \chi(k, l)$ iff $k < l$. Explain why your formula works.

NOTE: Your solutions must include enough detail to justify your conclusions.