MATH 445 – Homework 6

(*due Monday 31 October 2011*)

1. Using the rules in the notes (but not, of course, the Completeness Theorem) establish the following:

[4 pts] (a) $\vdash \exists x(P(x) \to Q(x)) \to (\forall x P(x) \to \exists x Q(x))$.

[4 pts] (b) $\vdash \forall x(P(x) \to \exists x P(x))$.

2. Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary relation symbol R. Let $\mathcal{A}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, <)$.

[3 pts] (a) Give a first-order sentence naturally expressing the following fact about $\mathcal{A}$: "Every number has an immediate predecessor."

[3 pts] (b) Give a formula $\varphi(x, y)$ of $\mathcal{L}$ such that $\mathcal{A} \models \varphi(k, l)$ iff $l = k + 2$. Explain why your formula works.

3. Let $\mathcal{L}$ be the language whose only non-logical symbol is a binary function symbol F. Let $\mathcal{B}$ be the $\mathcal{L}$-structure $(\mathbb{N}, +)$.

[3 pts] (a) Give a formula $\psi(x)$ of $\mathcal{L}$ which defines the set of odd numbers. Explain why your formula works.

[3 pts] (b) Give a formula $\chi(x, y)$ of $\mathcal{L}$ such that $\mathcal{B} \models \chi(k, l)$ iff $k < l$. Explain why your formula works.

NOTE: Your solutions must include enough detail to justify your conclusions.