MATH 445 – Homework 7

(*due Monday 23 November 2009*)

1. [4 pts] Give a formula $\varphi(x)$ of $\mathcal{L}_{\bar{\mathbb{N}}}$ which defines the set $X = \{2^0, 2^1, 2^2, \ldots, 2^k, \ldots\}$ in $\mathcal{N}$. Explain why your formula works.

2. [4 pts] Show that $X \subseteq \bar{\mathbb{N}}$ is definable in $\mathcal{N}$ iff K_X is definable in $\mathcal{N}$.

3. [4 pts] (a) Show that $\mathbf{PA} \models \forall x(0 \cdot x = 0)$.

[4 pts] (b) Show that $\mathbf{PA} \models \forall x \neg(x < x)$.

4. [4 pts] Assume that the binary relation R is representble in **PA** by the formula $\varphi(x, y)$. Define X as $\{k : R(k, l) \text{ holds for some } l\}$. Prove that $k \in X$ iff $\mathbf{PA} \models \exists y \varphi(x, y)$.

NOTE: Your solutions must include enough detail to justify your conclusions.