MATH 445 – Homework 7

(*due Monday 22 November 2010*)

1. [4 pts] Give a formula $\varphi(x)$ of $\mathcal{L}_{\mathbb{N}}$ which defines the set $X = \{3^k + 5^l | k, l \in \mathbb{N}\}$ in $\mathcal{N}$. Explain why your formula works.

2. [4 pts] Show that $X \subseteq \mathbb{N}$ is definable in $\mathcal{N}$ iff K_X is definable in $\mathcal{N}$.

3. [4 pts] (a) Show that if $k < n$ then $\mathbf{Q} \models \bar{k} < \bar{n}$.

Hint: Use induction on n to show "for every k if $k < n$ then $\mathbf{Q} \models \bar{k} < \bar{n}$."

[4 pts] (b) Show that $\mathbf{PA} \models \forall x \neg(x < x)$.

4. [4 pts] Assume that the binary relation R is definable in **PA** by the formula $\varphi(x, y)$. Define X as $\{k : R(k, l) \text{ holds for some } l\}$. Prove that X is also definable in **PA**.

NOTE: Your solutions must include enough detail to justify your conclusions.