MATH 445 – Homework 7

(*due Monday 21 November 2011*)

1. [5 pts] (a) Give a formula $\pi(x)$ of $\mathcal{L}_{\mathbb{N}}$ which defines the set of prime numbers in $\mathcal{N}$.

(b) Give a formula $\varphi(x)$ of $\mathcal{L}_{\mathbb{N}}$ which defines the set

$X = \{p^k \mid k \in \mathbb{N},\ p \text{ is prime}\}$ in $\mathcal{N}$. Explain why your formula works.

2. [5 pts] Show that $X \subseteq \mathbb{N}$ is definable in $\mathcal{N}$ iff K_X is definable in $\mathcal{N}$.

3. [5 pts] Show that $\mathbf{PA} \models \big(s(x) + y = s(x + y)\big)$. You can not assume the commutativity of addition – this is used in the proof of commutativity.

Hint: Use induction on y.

4. [5 pts] Assume that the binary relation R is definable in **PA** by the formula $\varphi(x, y)$. Define X as $\{k : R(k, l) \text{ holds for some } l\}$. Prove that X is also definable in **PA**. Explain carefully.

NOTE: Your solutions must include enough detail to justify your conclusions.