MATH 445 – Homework 8

(*due Friday 2 December 2011*)

1. [5 pts] Assume that the function $F : \mathbb{N} \to \mathbb{N}$ is recursive. Define G by $G(n) = F(0) + \cdots + F(n)$ for all $n \in \mathbb{N}$. Prove that G is recursive.

2. [5 pts] The *greatest common divisor* of k and l, $gcd(k, l)$, is defined to be the largest m which divides both k and l (unless $k = l = 0$, in which case $gcd(0, 0) = 1$). Show that gcd is recursive.

3. [5 pts] Assume that G_1 and G_2 are recursive unary functions and that P is a recursive subset of $\mathbb{N}$. Define F by $F(n) = G_1(n)$ if $P(n)$ holds, and $F(n) = G_2(n)$if $P(n)$ fails. Show that F is recursive.

4. [5 pts] Show that the function $C(k, i)$ defined in Theorem 3.1(c) is recursive.

NOTE: Your solutions must include enough detail to justify your conclusions.