MATH 445 – Homework 9

(*due Wednesday 9 December 2009*)

1. [4 pts] Define D by $D(0) = D(1) = 0$, $D(n) =$ the largest $k < n$ such that $k|n$ if $n > 1$. Prove that D is recursive.

2. [4 pts] Let $X = \{2^{2n+1} : n \in \bar{\mathbb{N}}\}$. Prove that X is recursive.

3. [4 pts] Prove that s, the successor function, is representable in **PA**. (Do not assume that $+$ is representable.)

4. [4 pts] Give an explicit definition of the relation $k! = l$ using the β-function.

5. [4 pts] Show that the function F defined by $F(n) = \ulcorner \overline{2n} \urcorner$ is recursive.

NOTE: Your solutions must include enough detail to justify your conclusions.