MATH 445 – Homework 9

(*due Friday 10 December 2010*)

1. [5 pts] Define the function H by $H(n) = [n/2] =$ the largest $k \leq n/2$. Prove that H is recursive.

2. [5 pts] Assume that $X \subseteq \mathbb{N}$ is recursive. Define the function F by $F(n) =$ the number of elements in X which are $< n$. Prove that F is recursive.

Hint: Use primitive recursion.

3. [5 pts] Let $X = \{0, 1, (1+2), (1+2+3), \ldots, (1+2+\cdots+n), \ldots\}$. Prove that X is recursive.

4. [5 pts] Show directly that the function F defined by $F(n) = \ulcorner\, \overline{n \cdot n}\, \urcorner$ is recursive.

Warning: $\overline{n \cdot n} \neq \bar{n} \cdot \bar{n}$.

NOTE: Your solutions must include enough detail to justify your conclusions.