MATH 445 – Homework 9

(*due Friday 9 December 2011*)

1. [5 pts] Define the function H by $H(n) = [n/2] =$ the largest $k \leq n/2$. Prove that H is recursive.

Warning: The function $F(n) = n/2$ is not recursive, since $n/2$ is not usually an integer.

2. [5 pts] Assume that $X \subseteq \mathbb{N}$ is recursive. Define the function F by $F(n) =$ the number of elements in X which are $< n$. Prove that F is recursive.

Hint: Use primitive recursion.

3. [5 pts] Let $X = \{2^{2n+1} \mid n \in \mathbb{N}\}$. Prove that X is recursive.

4. [5 pts] Assume that $g : \mathbb{N} \to \mathbb{N}$ is recursive and strictly increasing (that is, if $k < n$ then $g(k) < g(n)$). Let $Y = \{g(k) \mid k \in \mathbb{N}\}$ be the range of g. Prove that Y is recursive.

NOTE: Your solutions must include enough detail to justify your conclusions.