**MATH 446 – Exam I – 5 March 2010**

Work each numbered problem on a separate answer sheet. Show all your work for each problem clearly on the answer sheet for that problem. Write your name and the problem number on each answer sheet. Good luck.

**1.** [10 pts] (a) Let $f : X \to Y$. Let $A, B \subseteq Y$. Show that $f^{-1}\big[A \setminus B\big] = f^{-1}\big[A\big] \setminus f^{-1}\big[B\big]$.

[15 pts] (b) Show that $\big(A \to \mathcal{P}(A)\big) =_c \mathcal{P}(A \times A)$ for every set $A$.

**2.** [20 pts] Assume that there is a *surjection* $g$ of $A$ *onto* $B$. Prove that $B \leq_c \mathcal{P}(A)$.

**3.** [20 pts] Let $R$ be a set of ordered pairs. Define sets $A$ and $B$ such that $R \subseteq A \times B$, and **prove** that they are sets.

**4.** [20 pts] Let $\kappa, \lambda, \mu$ be cardinal numbers. Prove that $\kappa \leq_c \lambda$ implies that $\kappa^\mu \leq_c \lambda^\mu$.

**5.** [15 pts] Prove **directly from the definitions** that $S(0) \cdot m = m$ for all $m \in \mathbb{N}$. *In particular you cannot assume multiplication is commutative.*

**NOTE:** Your solutions must include enough detail to justify your conclusions.