

**MATH 446 – Exam I – 25 February 2011**

Work each numbered problem on a separate answer sheet. Show all your work for each problem clearly on the answer sheet for that problem. Write your name and the problem number on each answer sheet. Good luck.

1. [12 pts] (a) Let  $f : X \rightarrow Y$  be injective. Let  $A, B \subseteq X$ . Show that  $f[A \setminus B] = f[A] \setminus f[B]$ .  
[13 pts] (b) Let  $f : X \rightarrow Y$  be surjective. Let  $A \subseteq Y$ . Show that  $f[f^{-1}[A]] = A$ .
2. [10 pts] (a) Let  $A$  and  $B$  be sets such that  $A \cap B = \emptyset$ . Prove that  $\mathcal{P}(A \cup B) =_c \mathcal{P}(A) \times \mathcal{P}(B)$ .  
[15 pts] (b) Prove that  $(\mathbb{N} \rightarrow \mathbb{N}) =_c \mathcal{P}(\mathbb{N})$ .
3. [15 pts] Let  $A$  be a set of sets, and define  $B = \{\mathcal{P}(X) \mid X \in A\}$ . Prove that  $B$  is a set.
4. [20 pts] Let  $\kappa, \lambda, \mu$  be cardinal numbers. Prove that  $\kappa^{\lambda+\mu} =_c \kappa^\lambda \cdot \kappa^\mu$ .
5. [15 pts] Prove, by induction on  $n \in \mathbb{N}$ , that every surjection  $f : [0, n) \rightarrow [0, n)$  is 1-1.

**NOTE:** Your solutions must include enough detail to justify your conclusions.