MATH 446 – Exam II – 16 April 2010

Work each numbered problem on a separate answer sheet. Show all your work for each problem clearly on the answer sheet for that problem. Write your name and the problem number on each answer sheet. Good luck.

Important Note: No form of Choice is allowed on problems 1-3. DC only is allowed on problem 4. You may use any form of Choice given in class on problem 5 — be sure to state exactly what you are using and where you use it.

1. Let $\leq$ be the usual order on $\mathbb{N}$.

[10 pts] (a) Explicitly define a linear order $\leq^*$ on $\mathbb{N}$ which is not a well order. Justify your answer.

[15 pts] (b) Explicitly define a well order $\leq^{\#}$ on $\mathbb{N}$ such that $(\mathbb{N}, \leq) <_o (\mathbb{N}, \leq^{\#})$. Justify your answer.

2. [20 pts] Let U be a well ordered set. Prove, by induction, that the following holds for every $x \in U$:

there is some $z \in U$, where z is either 0_U or a limit point of U, and some $n \in \mathbb{N}$ such that $x = S^n(z)$.

S^n is defined by recursion on $\mathbb{N}$: $S^0(z) = z$, $S^{n+1}(z) = S\big(S^n(z)\big)$.

3. [20 pts] Assume that for every non-empty set A and for every equivalence relation E on A there is some function f whose domain is the set of equivalence classes of E and is such that for every $a \in A$, $f\big(\big[a/E\big]\big) \in \big[a/E\big]$. Prove that the Axiom of Choice holds.

Hint: It suffices to show the existence of choice sets.

4. [15 pts] Prove that $\mathbb{N} \leq_c A$ for every infinite set A using Dependent Choice, but not the full Axiom of Choice.

5. [20 pts] (AC) Assume that there is a surjection g of A onto B. Prove that $B \leq_c A$.

NOTE: Your solutions must include enough detail to justify your conclusions.