Math 446 — Exam 2 — Review Topics

Chapter 7. Well Ordered Sets.

w.o. sets, successor points, limit points, finite points, initial segments

similarities, $=_o$

expansive maps on p.o.sets

(proof by) transfinite induction

(definition by) transfinite recursion

initial similarities, $\leq_o$, comparability, well-foundedness

$\chi(A)$, Hartogs' Theorem

Fixed Point Theorem (7.35)

You are *not* responsible for the Iteration Lemma, the Least Fixed Point Theorem, and the proofs of Hartogs' Theorem and the Fixed Point Theorem.

Exercises x7.5, 7.11, 7.12, 7.25

Chapter 8. Choices.

the Axiom of Choice and its equivalence with the existence of choice sets and choice functions

the equivalence of AC with Cardinal Comparability and the Well Ordering Theorem

the equivalence of AC with existence of maximal chains and Zorn's Lemma

$\mathrm{AC}_{\mathbb{N}}$, DC, the implications between them and AC

DC implies that the well-founded p.o. sets are precisely those with no infinite descending chains

You are *not* responsible for graphs or DC for strings (8.14).

Exercises x8.1, 8.4

Chapter 9. Choice's Consequences.

$\mathbb{N}$ embeds into every infinite set

finite is equivalent to Dedekind-finite

(AC) the well-foundedness of $\leq_c$

(AC) existence of the next cardinal

(AC) absorption laws, cardinal minimum and supremum, and the results on infinite sums and products – through 9.22

You are *not* responsible for trees and the results about them (König's Lemma, the Fan Thm.), best well-orders, and cofinality; you are also not responsible for the proofs of the equality $\kappa \cdot \kappa = \kappa$ and of König's Theorem.

Exercises x9.7, 9.10, 9.15, 9.26