MATH 446 – Homework 1

(*due Friday 5 February 2010*)

1. [2 pts] Show that $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$.

2. Let $f : X \to Y$ and $A, B \subseteq X$.

[2 pts] (a) Show that $f[A \cap B] = f[A] \cap f[B]$ provided f is injective.

[1 pt] (b) Give an example to show that the equality in (a) can fail if f is not injective.

3. [3 pts] Let $f : X \to Y$ and $A, B \subseteq Y$. Prove that $f^{-1}[A \cap B] = f^{-1}[A] \cap f^{-1}[B]$.

4. In this problem the intervals are intervals in the real numbers. Explicitly define bijections establishing the following:

[2 pts] (a) $(\alpha, \beta) =_c (0, 1)$ for all reals $\alpha < \beta$.

[3 pts] (b) $(0, 1) =_c \mathbb{R}$.

[3 pts] (c) $[0, 1) =_c [0, 1]$.

5. [3 pts] Assume that A and B are disjoint. Show that $\mathcal{P}(A \cup B) =_c \mathcal{P}(A) \times \mathcal{P}(B)$.

6. [3 pts] Show that $(A \times B) \to C =_c (A \to (B \to C))$.

7. [3 pts] Show that $(A \to B) \leq_c \mathcal{P}(A \times B)$.

8. [5 pts] Prove that $(\mathbb{N} \to \mathbb{R}) =_c \mathbb{R}$.

NOTE: Your solutions must include enough detail to justify your conclusions.