MATH 446 – Homework 2

(*due Friday 19 February 2010*)

1. [2 pts] Give examples of non-empty sets A, B such that $\bigcup A = \bigcup B$ but $A \neq B$.

2. [3 pts] (a) Show that $\bigcup \mathcal{P}(A) = A$ for every set A.

[3 pts] (b) Show that $A \subseteq \mathcal{P}(\bigcup A)$ for every set A *all of whose elements are sets*.

Decide when equality holds.

3. [3 pts] Assume that every element of $\mathcal{A}$ is a subset of B. Show that $\bigcup \mathcal{A} \subseteq B$.

4. [4 pts] For any $\mathcal{A} \neq \emptyset$ we define $\bigcap \mathcal{A} = \{x | \ \forall A \in \mathcal{A}\ (x \in A)\}$. Prove from the axioms that for every $\mathcal{A} \neq \emptyset$, $\bigcap \mathcal{A}$ is a set.

5. For any sets A, B prove from the axioms that the following classes are sets.

[3 pts] (a) $\{\{x, y\} | x \in A, y \in B\}$.

[3 pts] (b) $\{\mathcal{P}(X) | X \subseteq A\}$.

6. [5 pts] A set I is *inductive* if $\emptyset \in I$ and for every $x \in I$ we also have $\{x\} \in I$. Prove that there is a smallest inductive set, that is an inductive set I_0 such that $I_0 \subseteq I$ for every inductive set I.

7. [4 pts] Define $\langle x, y \rangle$ to be $\{\{\emptyset, \{x\}\}, \{\{y\}\}\}$. Prove that $\langle x, y \rangle$ satisfies the properties (OP1) and (OP2) of ordered pairs.

NOTE: Your solutions must include enough detail to justify your conclusions.