MATH 446 – Homework 2

(*due Monday 13 February 2012*)

1. [5 pts] Show that $A \subseteq \mathcal{P}(\bigcup A)$ for every set A *all of whose elements are sets.*

State and prove a necessary and sufficient criterion for equality to hold.

2. [5 pts] For any $\mathcal{A} \neq \emptyset$ we define $\bigcap \mathcal{A} = \{x | \ \forall A \in \mathcal{A} \ (x \in A)\}$. Prove from the axioms that for every $\mathcal{A} \neq \emptyset$, $\bigcap \mathcal{A}$ is a set. Note: $\mathcal{A}$ could be a (proper) class.

3. For any sets A, B prove from the axioms that the following classes are sets.

[5 pts] (a) $\{\{x, y\} | x \in A, y \in B\}$.

[5 pts] (b) $\{\mathcal{P}(X) | X \subseteq A\}$.

4. [5 pts] A set I is *inductive* if $\emptyset \in I$ and for every $x \in I$ we also have $\{x\} \in I$. Prove that there is a smallest inductive set, that is an inductive set I_0 such that $I_0 \subseteq I$ for every inductive set I.

5. [5 pts] Define $\langle x, y \rangle$ to be $\{\{\emptyset, \{x\}\}, \{\{y\}\}\}$. Prove that $\langle x, y \rangle$ satisfies the properties (OP1) and (OP2) of ordered pairs.

NOTE: Your solutions must include enough detail to justify your conclusions.