MATH 446 – Homework 3

(*due Monday 1 March 2010*)

1. κ, λ, and μ are cardinal numbers as discussed in Chapter 4.

[4 pts] (a) Prove that $\kappa^{(\lambda+\mu)} =_c \kappa^\lambda \cdot \kappa^\mu$.

[4 pts] (b) Prove that $\left(\kappa^\lambda\right)^\mu =_c \kappa^{\lambda\cdot\mu}$.

[4 pts] (c) Prove that if $\lambda \leq_c \mu$ then $\kappa^\lambda \leq_c \kappa^\mu$ provided $\kappa \neq 0$.

Give a counterexample when $\kappa = 0$.

2. A set A is *Dedekind-infinite* provided there is an injection f of A into a *proper* subset of itself. A set which is not Dedekind-infinite is *Dedekind-finite*. Prove the following:

[4 pts] (a) If A is Dedekind-infinite and $A =_c B$ then B is also Dedekind-infinite.

[4 pts] (b) If B is Dedekind-finite and $A \subseteq B$ then A is Dedekind-finite.

[5 pts] (c) If A is inductive then A is Dedekind-infinite.

3. [5 pts] Let $(\mathbb{N}, 0, S)$ be a Peano-system and let $+$ be defined on $\mathbb{N}$ by recursion as in Chapter 5. Prove that

$$\text{for all } k, l, n \in \mathbb{N} \text{ if } k + n = l + n \text{ then } k = l.$$

Hint: let $X = \{n \in \mathbb{N} | \forall k \forall l (k + n = l + n \rightarrow k = l)\}$ and use Induction.

NOTE: Your solutions must include enough detail to justify your conclusions.