MATH 446 – Homework 3

(*due Monday 20 February 2012*)

1. κ, λ, and μ are cardinal numbers as discussed in Chapter 4.

[4 pts] (a) Prove that $(\kappa \cdot \lambda)^{\mu} =_c \kappa^{\mu} \cdot \lambda^{\mu}$.

[5 pts] (b) Prove that $\left(\kappa^{\lambda}\right)^{\mu} =_c \kappa^{\lambda \cdot \mu}$.

[5 pts] (c) Prove that if $\lambda \leq_c \mu$ and $\kappa \neq 0$ then $\kappa^{\lambda} \leq_c \kappa^{\mu}$.

[1 pt] Give a counterexample when $\kappa = 0$.

2. A set A is *Dedekind-infinite* provided there is an injection f of A into a *proper* subset of itself. A set which is not Dedekind-infinite is *Dedekind-finite*. Prove the following:

[5 pts] (a) If A is Dedekind-infinite and $A \leq_c B$ then B is also Dedekind-infinite.

[5 pts] (b) If A is inductive then A is Dedekind-infinite.

3. [5 pts] Let $(A, 0, S)$ be a Peano-system. Prove that there is a set $B \subseteq A$ such that $0 \in B$ and for every $b \in B$, $S(b) \notin B$, $S(S(b)) \notin B$, but $S(S(S(b))) \in B$. You may use the definition of Peano-systems and the Recursion Theorem, but nothing after that.

NOTE: Your solutions must include enough detail to justify your conclusions.