MATH 446 – Homework 4

(*due Friday 26 March 2010*)

1. [6 pts] Prove that every linear order of a finite set A is a well order.

Hint: Prove by induction on $\#(A)$.

2. Define the lexicographic order $\leq_l$ on $\mathbb{N} \times \mathbb{N}$ as follows:

$(k, n) \leq_l (k', n')$ iff either $k < k'$ or $[k = k'$ and $n \leq n']$.

[6 pts] (a) Prove that $U = (\mathbb{N} \times \mathbb{N}, \leq_l)$ is a well ordered set.

[4 pts] (b) Explicitly identify 0_U, S_U, and the limit points of U.

3. Let U be an infinite well ordered set.

[8 pts] (a) Prove that there is an order-preserving bijection between $(\mathbb{N}, \leq)$ and the initial segment of U consisting of all the finite points of U.

[6 pts] (b) Let a be an infinite point of U. Prove that there is a *largest* limit point $b \in U$ such that $b \leq_U a$.

NOTE: Your solutions must include enough detail to justify your conclusions.