MATH 446 – Homework 4

(*due Friday 11 March 2011*)

1. [6 pts] Prove that every linear order of a finite set A is a well order. Use induction on $\#(A)$.

2. Define the lexicographic order $\leq_l$ on $\mathbb{N} \times \mathbb{N}$ as follows:

$(k, n) \leq_l (k', n')$ iff either $k < k'$ or $[k = k' \text{ and } n \leq n']$.

[6 pts] (a) Prove that $U = (\mathbb{N} \times \mathbb{N}, \leq_l)$ is a well ordered set.

[4 pts] (b) Explicitly identify 0_U, S_U, and the limit points of U.

3. Let U be a well ordered set.

[6 pts] (a) Let a be an infinite point of U. Prove that there is a *largest* limit point $b \in U$ such that $b \leq_U a$.

[8 pts] Prove that every $y \in U$ can be expressed as $y = S^n(x)$ where x is either 0 or a limit point, $n \in \mathbb{N}$, and $S^n(x)$ is the function of two variables, n and x, defined by the recursion $S^0(x) = x$, $S^{n+1}(x) = S(S^n(x))$.

NOTE: Your solutions must include enough detail to justify your conclusions.