MATH 446 – Homework 4

(*due Monday 12 March 2012*)

1. [6 pts] Prove that every linear order of a finite set A is a well order. Use induction on $\#(A)$.

2. [8 pts] Define the lexicographic order $\leq_l$ on $\mathbb{N} \times \mathbb{N}$ as follows:

$(k, n) \leq_l (k', n')$ iff either $k < k'$ or $[k = k'$ and $n \leq n']$.

Prove that $U = (\mathbb{N} \times \mathbb{N}\ ,\ \leq_l)$ is a well-ordered set. (First, of course, you must prove it is linearly ordered.)

3. [10 pts] Let U be a well ordered set. Prove that every $y \in U$ can be expressed *uniquely* as $y = S^n(x)$ where x is either 0 or a limit point, $n \in \mathbb{N}$, and $S^n(x)$ is the function of two variables, n and x, defined by the recursion $S^0(x) = x$, $S^{n+1}(x) = S(S^n(x))$.

4. [6 pts] Let $\left[\mathbb{N}\right]^2 = \{X \subseteq \mathbb{N} : \#(X) = 2\}$. Define a well-ordering of $\left[\mathbb{N}\right]^2$ and prove it is a well order.

NOTE: Your solutions must include enough detail to justify your conclusions.