MATH 446 – Homework 5

(*due Friday 2 April 2010*)

1. [5 pts] Let U be a well ordered set, let $x, y \in U$, and assume that $x <_U y$ and there is no limit point $l \in U$ with $x <_U l \leq_U y$. Prove that $y = S^n(x)$ for some $n \in \mathbb{N}$.

($S^n(x)$ is the function of the two variables n and x defined by the recursion $S^0(x) = x$, $S^{n+1}(x) = S(S^n(x))$, where S is S_U.)

2. [5 pts] Let κ be an infinite well orderable cardinal number. Prove that $\kappa + 1 =_c \kappa$.

3. [5 pts] Prove that for every set A there is a well ordered set V such that there is no surjection π of A onto V.

4. [5 pts] Prove that if $A \leq_c B$ then $\chi(A) \leq_o \chi(B)$.

5. [5 pts] Prove that $\chi\big([0, n)\big) =_o [0, n + 1)$.

6. [5 pts] Let U be a well ordered set. Prove that if $U \leq_c A$ then $U <_o \chi(A)$.

NOTE: Your solutions must include enough detail to justify your conclusions.