MATH 446 – Homework 5

(*due Monday 28 March 2011*)

1. [5 pts] Define $[\mathbb{N}]^2 = \{X \subseteq \mathbb{N} |\ \#(X) = 2\}$. Define a well order of $[\mathbb{N}]^2$, and prove it is a well order.

2. Let A be a well orderable set.

[5 pts] (a) Assume that A is infinite, and let a^* be some object not in A. Prove that $(A \cup \{a^*\}) =_c A$.

[5 pts] (b) Assume that B is also well orderable. Prove that $(A \cup B)$ is well orderable.

3. [5 pts] Prove that for every set A there is a well ordered set V such that there is no surjection π of A onto V.

4. [5 pts] Prove that if $A \leq_c B$ then $\chi(A) \leq_o \chi(B)$.

5. [5 pts] Let U be a well ordered set. Prove that if $U \leq_c A$ then $U <_o \chi(A)$.

NOTE: Your solutions must include enough detail to justify your conclusions.