MATH 446 – Homework 7

(*due Friday 22 April 2011*)

1. [6 pts] Prove that the Separation Axiom (III) follows from the other axioms in the first group (I, II, IV, V) together with the Axiom of Replacement (VIII).

2. [6 pts] Let F be a unary definite operation. Prove that for every set A there is a smallest (under $\subseteq$) set $\bar{A}$ such that $A \subseteq \bar{A}$ and $\bar{A}$ is closed under F (i.e., if $x \in \bar{A}$ then $F(x) \in \bar{A}$).

3. [6 pts] Assume that A is a non-empty set of transitive sets. Prove that $\bigcap A$ and $\bigcup A$ are both transitive.

4. [6 pts] Prove that a set A is grounded iff $TC(A)$ is grounded.

5. [6 pts] Prove that a set a is ill-founded iff there is some set A such that $a \in A$ and $\forall x \in A\ \exists y \in A\ (y \in x)$.

NOTE: Your solutions must include enough detail to justify your conclusions.