MATH 446 – Homework 8

(*due Friday 7 May 2010*)

Note: α, β, and γ are all ordinals.

1. [6 pts] Prove that $\alpha + \beta < \alpha + \gamma$ iff $\beta < \gamma$.

2. [5 pts] (a) Prove that for every $\gamma \geq \alpha$ there is exactly one β such that $\gamma = \alpha + \beta$.

[4 pts] (b) Prove that for every $\gamma \geq \beta$ there is some α such that $\gamma = \alpha + \beta$.

[1 pt] (c) Give an example to show that uniqueness can fail in (b).

3. A definite operation $F : ON \to ON$ is *normal* if $\alpha < \beta$ implies $F(\alpha) < F(\beta)$ and for every limit ordinal ξ, $F(\xi) = \bigcup\{F(\alpha) | \alpha < \xi\}$.

[5 pts] (a) Let F be a normal operation. Prove that $\alpha \leq F(\alpha)$ for all α and that $F(\xi)$ is a limit ordinal for every limit ordinal ξ.

[3 pts] (b) Assume that F and G are both normal operations. Prove that $H(\alpha) = F(G(\alpha))$ is also normal.

4. [6 pts] Prove that $\text{Rank}(\alpha) = \alpha$ for every α.

NOTE: Your solutions must include enough detail to justify your conclusions.