MATH 446 – Homework 8

(*due Monday 30 April 2012*)

Note: α, β, and γ are all ordinals.

1. [6 pts] Prove, using only the Zermelo Axioms, that the Axiom of Replacement is equivalent to the following:

for every definite operation F and every set A there is some set B such that $A \subseteq B$ and $F[B] \subseteq B$.

2. [6 pts] Prove that $\alpha + \beta < \alpha + \gamma$ iff $\beta < \gamma$.

3. [6 pts] (a) Prove that for every $\gamma \geq \alpha$ there is exactly one β such that $\gamma = \alpha + \beta$.

[3 pts] (b) Give examples (with proof) of ordinals α_0, α_1, and β such that $\alpha_0 + \beta = \alpha_1 + \beta$ but $\alpha_0 \neq \alpha_1$.

4. A definite operation $F : ON \to ON$ is *normal* if $\alpha < \beta$ implies $F(\alpha) < F(\beta)$ and for every limit ordinal ξ, $F(\xi) = \bigcup\{F(\alpha) | \alpha < \xi\}$.

[6 pts] Prove that for every normal operation F the following hold:

$\alpha \leq F(\alpha)$ for all α and $F(\xi)$ is a limit ordinal for every limit ordinal ξ.

[3 pts] Assume that F and G are both normal. Prove that $H(\alpha) = F(G(\alpha))$ is also normal.

NOTE: Your solutions must include enough detail to justify your conclusions.