MATH 446 – Homework 9

(*due Wednesday 9 May 2012*)

1. Let $F : ON \to ON$ be a normal operation. Prove that for every α there is some $\gamma \geq \alpha$ with $F(\gamma) = \gamma$.

2. Prove that for every $\alpha \in ON$ there is some $\gamma \geq \alpha$ such that for all $\xi < \gamma$ we have $\xi + \gamma = \gamma$.

3. We have defined $cf(\alpha)$, whenever α is a limit ordinal, as the least β such that there is an order-preserving injection of β into α with $\alpha = \bigcup\{f(\xi) \mid \xi < \beta\}$. Prove that for infinite cardinals κ, $cf(\kappa)$ is the least cardinal λ such that $\kappa = \bigcup\{X_i \mid i \in \lambda\}$ for some sets $|X_i| < \kappa$.

4. Prove that for every $N \in \omega$, every $\alpha < \omega^N$ can be written uniquely as $\omega^n \cdot k + \beta$ where $n < N$, $k \in \omega$, and $\beta < \omega^n$.

NOTE: Your solutions must include enough detail to justify your conclusions.