# MODEL COMPLETE THEORIES: OUTLINE

**Definition 1.** A theory $T$ of $L$ is model complete iff for all $\mathfrak{A}, \mathfrak{B}$ models of $T$ if $\mathfrak{A} \subseteq \mathfrak{B}$ then $\mathfrak{A} \prec \mathfrak{B}$.

**Lemma 1.** *$T$ is model complete iff for every $\mathfrak{A} \models T$ the set $(T \cup \Delta_{\mathfrak{A}})$ axiomatizes a complete theory of $L(A)$.*

**Definition 2.** If $X \subseteq A$ then $\Delta_{\mathfrak{A}}^{X} = (\Delta_{\mathfrak{A}} \cap Sn_{L(X)})$.

**Theorem 1.** *Assume that for every model $\mathfrak{A}$ of $T$ and for every finite $X \subseteq A$ the set $(T \cup \Delta_{\mathfrak{A}}^{X})$ axiomatizes a complete theory of $L(X)$. Then $T$ is model complete.*

*Proof.* Let $\sigma \in Sn_{L(A)}$. Then in fact $\sigma \in Sn_{L(X)}$ for some finite $X \subseteq A$. So by hypothesis either $\sigma$ or $\neg\sigma$ is a consequence of $(T \cup \Delta_{\mathfrak{A}}^{X})$, and thus also a consequence of $(T \cup \Delta_{\mathfrak{A}})$. $\square$

In fact the hypothesis of the Theorem implies that $T$ has *quantifier elimination* (is *q.e.*), that is, every formula of $L$ is equivalent to some open formula on all models of $T$.

The three theories earlier shown to be complete using the Łoś-Vaught test can be shown to be model complete using this Theorem via essentially the same arguments.