AMSC/CMSC 460: HW #11
Do not submit

Note: All integration problems should be done as Gaussian integration.

1. Find a formula of the form

$$\int_{-\infty}^{\infty} f(x)e^{-x^2}dx \approx A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2)$$

that is exact for all polynomials of degree 5.

2. Find a formula of the form

$$\int_0^{\infty} f(x)e^{x}dx \approx A_0 f(x_0) + A_1 f(x_1).$$

that is exact for all polynomials of degree 3. Hint: Use Laguerre polynomials.

3. Find a formula of the form

$$\int_0^1 xf(x)dx \approx A_0 f(x_0) + A_1 f(x_1).$$

that is exact for all polynomials of degree 3.

4. Find a formula of the form

$$\int_0^1 x^2 f(x)dx \approx A_0 f(x_0) + A_1 f(x_1).$$

that is exact for all polynomials of degree 3.

5. Let L be an exact quantity that is approximated by $D(h)$ such that

$$L = D(h) + a_1 h + a_3 h^3 + a_5 h^4 + \dots$$

Use Richardson's extrapolation to obtain a third-order approximation of L. Repeat the process and use Richardson's extrapolation to obtain a fourth-order approximation of L. (Note that if the approximated quantity was an integral, we would call the process Romberg's integration instead of Richardson's extrapolation, but they really are the same).

6. Let I be an exact quantity that is approximated by $A(h)$ such that

$$I = A(h) + a_1\sqrt{h} + a_2 h + a_3 h^{3/2} + \dots.$$

Use Richardson's extrapolation to find a first order approximation to I. Repeat the process to find an approximation of order 3/2.