AMSC/CMSC 460: HW #2
Due: Thursday 2/8/18 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Perform five iterations of Newton's method for finding a root of $p(x) = x^3 - 5x^2 + 3x - 7$ starting with $x_0 = 5$.

2. Write a Matlab program to solve the equation $x = \tan x$ by means of Newton's method. Find the roots closest to 4.5 and 7.7.

3. Use the bisection method to compute a positive root of $x^2 - 4x \sin x + (2\sin x)^2 - 1 = 0$ accurate to 0.01.

4. The function
$$f(x) = \frac{x}{\sqrt{1+x^2}}$$
has a unique root $f(x) = 0$ only for $x = 0$.
   (a) Show that Newton's method gives $x_{k+1} = -x_k^3$. Conclude that the method succeeds if and only if $|x_0| < 1$.
   (b) Draw graphs to illustrate the first few iterates when $x_0 = .25$, $x_0 = .5$, and $x_0 = 1.5$.

5. Let p be a positive number. What is the value of the following expression?
$$x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}$$
Hint: observe that x can be written as the limit of a sequence for which the elements are defined as $x_{n+1} = \sqrt{p + x_n}$.

6. Let $p > 1$. What is the value of the following continued fraction?
$$x = \frac{1}{p + \frac{1}{p + \frac{1}{p + \cdots}}}$$

7. Write down two different fixed-point procedures for finding a zero of the function $f(x) = 2x^2 + 6e^{-x} - 4$.

8. Show that the following method can be used for computing $\sqrt{R}$:
$$x_{n+1} = \frac{x_n(x_n^2 + 3R)}{3x_n^2 + R}.$$