AMSC/CMSC 460: HW #4
Due: Thursday 2/22/18 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Compute the infinity norm and the condition number in the infinity norm for the following matrices: (you may use matlab to compute A^{-1})

$$A = \begin{pmatrix} -1 & 0.1 & 0.05 \\ 0.1 & 1.1 & 0.1 \\ 0.05 & -0.1 & 0.9 \end{pmatrix},$$

$$A = \begin{pmatrix} 2 & 2.2 & 1 \\ 2 & 2 & 1 \\ 1.9 & 2.1 & 0.9 \end{pmatrix}.$$

2. Let $f(x) = -2x^5$. Find the second Taylor polynomial $P_2(x)$ about $x_0 = 0$.

3. Let $f(x) = \sqrt{x+1}$. Find the third Taylor polynomial $P_3(x)$ about $x_0 = 0$. Use $P_3(x)$ to approximate $\sqrt{0.45}$, $\sqrt{0.8}$, $\sqrt{1.1}$, and $\sqrt{1.4}$ Determine the actual error of these approximations.

4. The Maclaurin series for $(1+x)^n$ is also known as the binomial series. It states that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots, \qquad (x^2 < 1).$$

   Derive this series by computing a Taylor's for $(1+x)^n$ around $x = 0$. Note that it is not assumed that n is an integer. Give its particular form in summation notation for $n = \frac{1}{2}$. Use this expression to approximate $\sqrt{1.0001}$.

5. Read Chapters 2 and 3 in Michael Overton's book "Numerical Computing with IEEE Floating Point Arithmetic". Solve problems 3.1, 3.2, 3.3, 3.4, 3.6, 3.8. These chapters can be downloaded from the university library's webpage.