

AMSC/CMSC 460: Midterm 1

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**Read carefully the following instructions:**

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

**Problems: (Each problem = 10 points)**

1. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 20 & 30 \\ 3 & 30 & 94 \end{pmatrix}$$

- (a) Find a Cholesky decomposition for  $A$ .
- (b) Explain how the Cholesky decomposition can be used to easily solve a linear system  $Ax = b$ .
2. Let  $f(x) = x^3 - 2x - 3$ .
- (a) Use the continuity of  $f(x)$  to explain why  $f(x)$  has at least one positive root.
- (b) Write Newton's method for finding a root of  $f(x)$ . Compute two iterations of the method starting from  $x_0 = 2$ .
- (c) Write the Secant method for finding a root of  $f(x)$ . Compute one iteration of the method starting from  $x_0 = 0$  and  $x_1 = 2$ .
3. (a) Write the number 14.42 in base 2. (Compute the first 10 digits after the binary point).
- (b) Explain two possible approaches for representing the number -17 on a computer with an 8-bit word.
4. Let  $f(x) = x^4 - 3x^2 + 3$ .  
Let  $x_0 = -1, x_1 = 1, x_2 = 2$ , and let  $y_j = f(x_j)$  for  $j = 0, 1, 2$ .
- (a) Write Newton's form for the interpolation polynomial that interpolates the data at the given points.
- (b) Write Lagrange's form for the interpolation polynomial that interpolates the data at the given points.
- (c) Assume that in addition to  $x_0, x_1, x_2$ , you are given one additional interpolation point  $x_3 = 0$ . Using the divided differences notation, write the term that should be added to the interpolation polynomial from part (a) in order to obtain a new polynomial that interpolates the data at all four points  $x_0, \dots, x_3$ .