

AMSC/CMSC 460: Midterm 2

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**Read carefully the following instructions:**

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

**Problems: (Each problem = 10 points)**

1. (a) Write the Newton form of the interpolation polynomial of degree  $\leq 3$  that

interpolates: 
$$\frac{x}{f(x)} \left\| \begin{array}{c|c|c|c} 0 & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0 \end{array} \right.$$

- (b) Use the divided differences with repetitions notation to write the Hermite polynomial that interpolates the data:

$$f(0) = 1, f'(0) = 0, f(1) = 0, f'(1) = 0.$$

2. (a) Determine all the values of  $a, b, c, d, e, f$  for which the following function is a cubic spline

$$f(x) = \begin{cases} ax + b(x-1)^3, & x \in (-\infty, 0], \\ cx + d(x-1)^3, & x \in [0, 1], \\ ex + f(x-1)^3, & x \in [1, \infty). \end{cases}$$

- (b) Determine the values of the parameters so that the cubic spline from part (a) interpolates

$$\frac{x}{f(x)} \left\| \begin{array}{c|c|c} 0 & 1 & 2 \\ \hline -2 & 2 & 7 \end{array} \right.$$

3. (a) Let  $w(x) = e^x$ . Find the first two orthogonal polynomials with respect to the inner product

$$\langle f(x), g(x) \rangle_w = \int_0^1 f(x)g(x)w(x)dx.$$

(Do not normalize the polynomials).

- (b) Normalize the polynomial of degree zero you found in part (a).  
 (c) Use the orthogonal polynomials you found in part (a) to find the polynomial of degree  $\leq 1$ ,  $p_1(x)$ , that minimizes

$$\int_0^1 e^x(e^{-x} - p_1(x))^2 dx.$$

In solving both parts of this problem you may use the following formula:

$$\int x^n e^{ax} dx = \frac{e^{ax}}{a} \left( x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \dots - \frac{(-1)^n n!}{a^n} \right) + c, \quad n = \text{positive integer}$$