AMSC/CMSC 460: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 10 points)

1. (a) Write the number 35.35 in base 2. (Compute the first 10 digits after the binary point).
   (b) Explain how 35.35 can be represented as a floating point number on a 64-bit computer.
   (c) Explain how 35.35 can be stored with a fixed point representation on a 64-bit computer. What are the advantages of a floating point representation over a fixed point representation?
   (d) Explain two approaches for representing the (negative) number -35 on a computer with a 64-bit word.

2. Consider the following matrix A, and its inverse A^{-1}:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 3 \\ 0 & 2 & -1 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -1/6 & 1/6 & 2/3 \\ -1/3 & 1/3 & 1/3 \end{pmatrix}$$

   (a) Compute the condition number of A in the infinity norm.
   (b) Find an LU decomposition of A where L is a unit lower triangular matrix.
   (c) Use the LU decomposition that you found, to solve $Ax = b$ with $b = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$.

3. Let $f(x) = e^{-x} - x^2$.
   (a) Prove that there exists at least one point $x^* \in [0, 10]$ for which $f(x^*) = 0$.
   (b) Starting from $x_0 = 0$, use Newton's method to compute two approximations x_1 and x_2 for a root of $f(x)$.
   (c) Starting from $x_0 = 0$ and $x_1 = 1$, compute one iteration of the secant method for the given function $f(x)$.

4. Let $f(x) = \cos(\pi x)$.
   Let $x_0 = -1, x_1 = 0, x_2 = 1$, and let $y_j = f(x_j)$ for $j = 0, 1, 2$.
   (a) Write Newton's form for the interpolation polynomial, $P_2(x)$, that interpolates the data at the three given points.
   (b) Write Lagrange's form for the interpolation polynomial, $P_2(x)$, that interpolates the data at the three given points.
   (c) Verify that the answers to parts (a) and (b) are identical. Explain the advantages of Newton's form over Lagrange's form.
   (d) Write the Lagrange form of the polynomial $p_3(x)$, that interpolated $f(x) = \sin(\pi x)$ at the four points: $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 1/2$.